FSBday:

Implementing Wagner's Generalized Birthday Attack against the round-1 SHA-3 Candidate FSB

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Wagner's generalized birthday attack

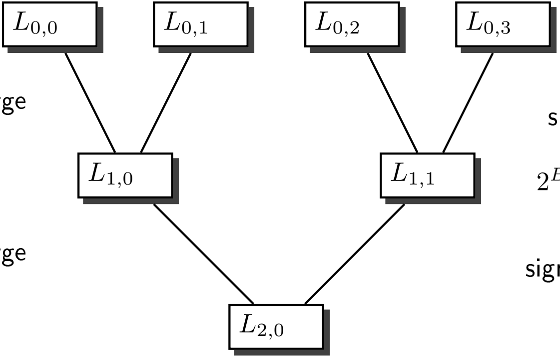
Given 2^{i-1} lists containing B-bit strings.

Generalized birthday problem:
The 2^{i-1}-sum problem consists of finding 2^{i-1} elements — exactly one per list — such that their sum equals 0 (bitwise modulo $2 \Rightarrow$ xor).

Wagner (CRYPTO '02)

We can expect a solution to the generalized birthday problem after one run of an algorithm using time $O((i-1) \cdot 2^{B/i})$ and lists of size $O(2^{B/i})$.

Wagner's tree algorithm



$2^{B/3}$ elements

1. merge

compare on least significant $B/3$ bits

$2^{B/3}$ elements of xors

2. merge

compare on least significant $2 \times B/3$ bits

Expect to get 1 match after the last merge step.

Tree algorithm for 2^{i-1} lists

The tree algorithm generalizes to 2^{i-1} lists as follows:

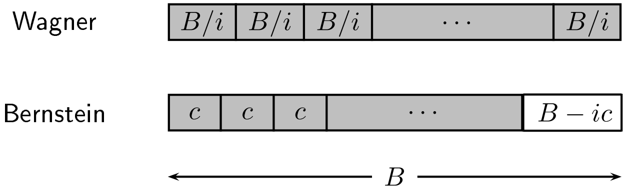
- Compare lists — always two at a time — by looking at the least significant B/i bits of elements.
- On level $i-2$ we are left with two lists whose elements need to be compared on $2B/i$ remaining bits.

Precomputation step

Suppose that there is space for lists of size only 2^c with $c < B/i$.

Bernstein (SHARCS '07):

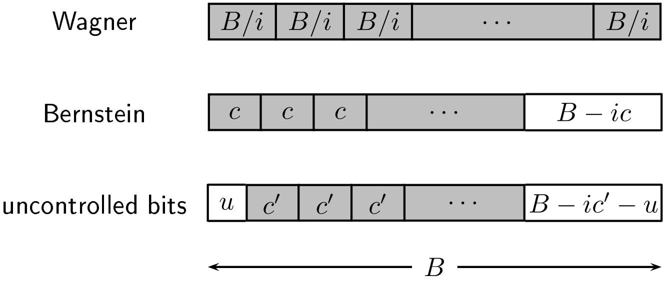
- Generate $2^{c \cdot (B-ic)}$ entries and only consider those of which the least significant $B - ic$ bits are zero.
- Then apply Wagner's algorithm with lists of size 2^c and clamp away c bits on each level.



Repeating (parts of) the tree algorithm

- When performing the algorithm with smaller lists, u bits remain "uncontrolled" at the end.

- Deal with the lower success probability by repeatedly running the attack with different clamping values.

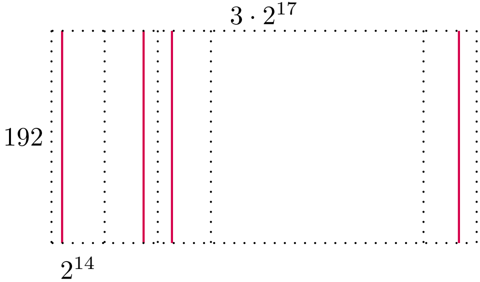


Target: the compression function of FSB_{48}

Given a binary random 192×393216 matrix H; number of blocks: $w = 24$.

Input: a regular weight-24 bit string of length 393216, i.e., there is exactly a single 1 in each interval $[(i-1) \cdot 16384, i \cdot 16834]_{1 \leq i \leq 24}$.

Output: Xor the 24 columns indicated by the input bit string.



Goal: Find a collision in FSB_{48}'s compression function; i.e., find 48 columns — exactly 2 per block — which add up to 0.

Applying Wagner to FSB_{48}

Determine the number of lists for a Wagner attack on FSB_{48}.

- We choose 16 lists to solve this particular 48-sum problem. (16 is the highest power of 2 dividing 48).
- Each list entry will be the xor of three columns coming from one and a half blocks (no overlaps!).
  $\rightarrow$ We can generat at most 2^{40} elements per list.

Straightforward Wagner

- Applying Wagner's attack with 16 lists in a straightforward way means that we need to have at least $2^{\lceil 192/5 \rceil}$ entries per list.
- By clamping away 39 bits in each step we expect to get at least one collision after one run of the tree algorithm.

List entries

- Reduce amount of data by clamping away 2 bits $\Rightarrow 2^{38}$ entries per list (clamp 38 bits on each level)
- Ultimately we are not interested in the value of the entry; but in the column positions in the matrix that lead to this all-zero value.
  - Value-only representation
  - Positions-only representation: keep full positions; if we we need the value (or parts of it) it can be dynamically recomputed from the positions.
- Note: Unlike storage requirements for values the number of bytes for positions increases with increasing levels.

Storing positions

- Encode column positions of each entry in 40 bits (5 bytes) for the first level.
- The expected number of entries per list remains the same but the number of lists halves; so the total amount of data is the same on each level when using dynamic recomputation.
- Storing 16 lists with 2^{38} entries, each entry encoded in 5 bytes requires 20480 GB of storage space.
- The Coding and Cryptography Computer Cluster at Eindhoven University of Technology only has a total hard disk space of about 5440 GB, so we have to adapt our attack strategy to this limitation.

Adapt attack strategy

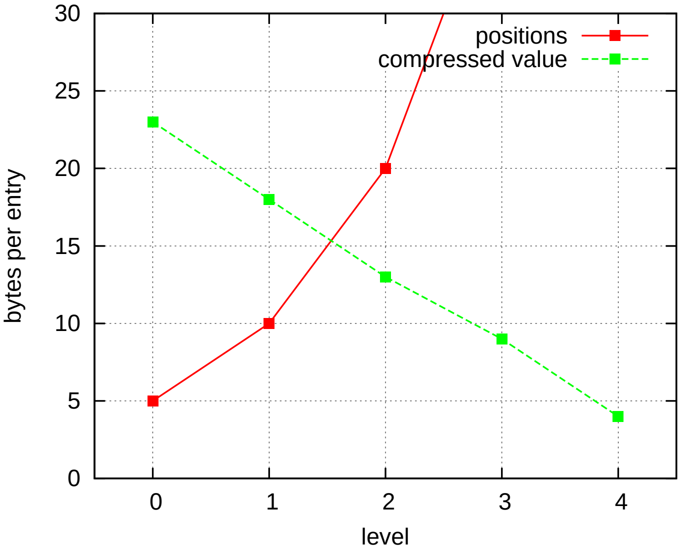
- Can handle at most $5\ TB/16\ lists/5 = 2^{36}$ entries per list.
- A straightforward implementation would use lists of size 2^{36}: clamp 4 bits during list generation; this leads to 2^{36} values for each of the 16 lists (as we can generate at most 2^{40} elements per list).
- We expect to run the attack 256.5 times until we find a collision.

Attack in two phases

Idea

- First phase: Figure out which clamping constants yield collision
- Second phase: Compute matrix positions yielding collision
- During phase one we do not have to store positions of entries
- On each level compress entries to shortest possible representation

Attack in two phases

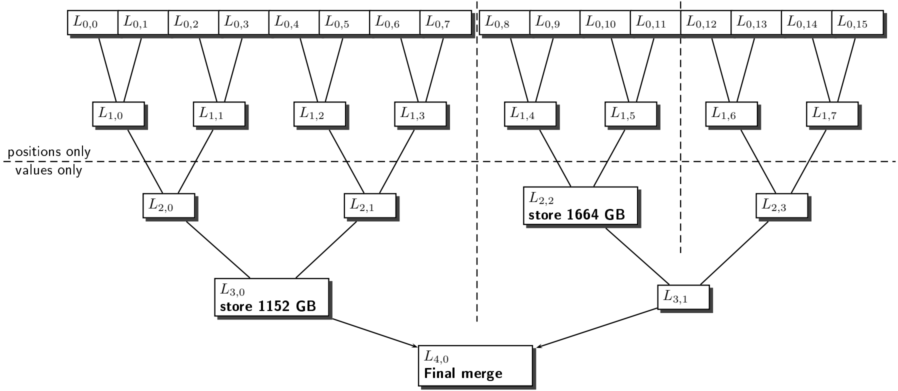


Attack in two phases

Idea

- First phase: Figure out which clamping constants yield collision
- Second phase: Compute matrix positions yielding collision
- During phase one we do not have to store positions of entries
- On each level compress entries to shortest possible representation
  - Level 0: 5 bytes (positions only)
  - Level 1: 10 bytes (positions only)
  - Level 2: 13 bytes (values only)
  - Level 3: 9 bytes (values only)
- Use lists of size 2^{37}
- Clamp 3 bits through precomputation
- This leaves 4 bits "uncontrolled" $\rightarrow$ 16.5 repetitions expected

Attack Strategy



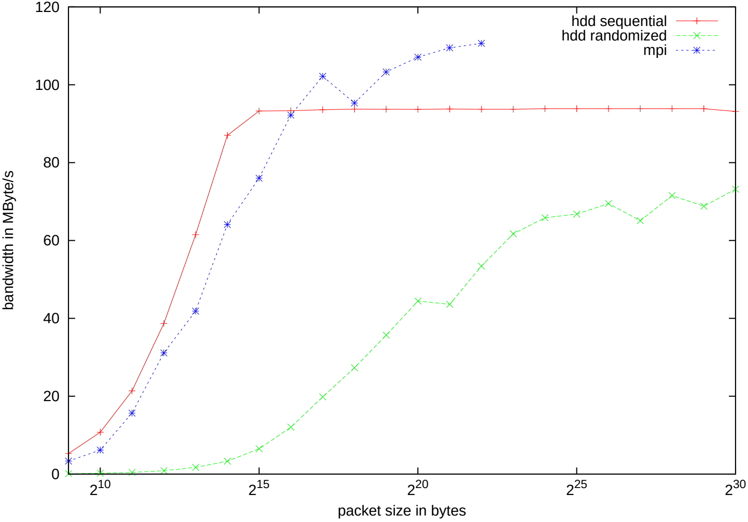
$$\Longrightarrow 1152\text{ GB} + 1664\text{ GB} + 2560\text{ GB} = 5376\text{ GB}$$

Our hardware

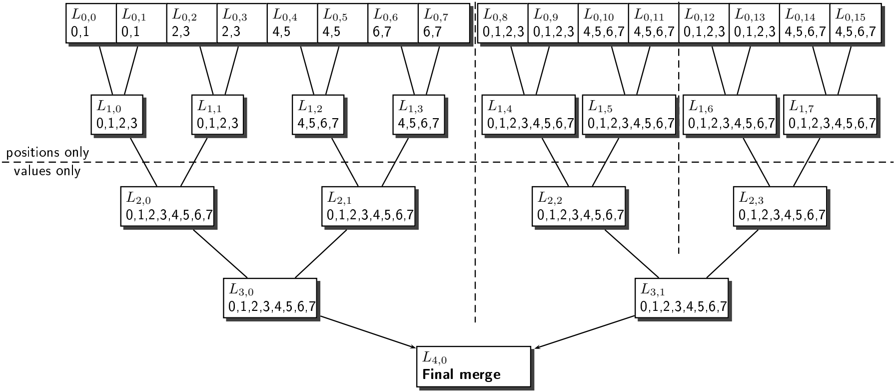
Cluster of 8 nodes:

- Intel Core 2 Quad Q6600 CPU, 2.40 GHz
- 8 GB of RAM per node
- about 680 GB accessible mass storage
- connected via switched Gigabit Ethernet

Finding the bottleneck(s)



Parallelization



Parallelization

- Split fractions further into 512 parts of 640 MB each (presort, according to 9 bits)
- Sort and merge parts independently in memory
- Pipeline
  - Loading from hard disk into memory
  - Sorting of two parts
  - Merging of previously sorted parts
- Requires 6 parts in memory at the same time (3.75 GB)

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- Two blocks of operations:
  - Load, Sort, Merge, Send
  - Receive, Presort, Store

Timing Results

- Timings for phase 1:
  - Computation of list $L_{3,0}$: ~ 32 h (once)
  - Computation of list $L_{2,2}$: ~ 14 h (once)
  - Computation of list $L_{2,3}$: ~ 14 h (exp. 16.5×)
  - Computation of list $L_{3,1}$: ~ 4 h (exp. 16.5×)
  - Check for collision in $L_{3,0}$ and $L_{3,1}$: ~ 3.5 h (exp. 16.5×)
- Expected time for phase 1: $32 + 14 + 16.5 \cdot (14 + 4 + 3.5) =$ 400.7 h or 17 days
- Time for phase 2: ∼33 h per half-tree, in total ∼66 h
- Expected time in total: ∼19.5 days.

Result

We already found a solution in step one after only five iterations!

In total the attack took 7 days, 23 hours and 53 minutes.

The result:

734, 15006, 20748, 25431, 33115, 46670, 50235, 51099, 70220, 76606, 89523, 90851, 99649, 113400, 118568, 126202, 144768, 146047, 153819, 163606, 168187, 173996, 185420, 191473 198284, 207458, 214106, 223080, 241047, 245456, 247218, 261928, 264386, 273345, 285069, 294658, 304245, 305792, 318044, 327120, 331742, 342519, 344652, 356623, 364676, 368702, 376923, 390678

Further information

Paper: http://eprint.iacr.org/2009/292

Cluster: http://www.win.tue.nl/cccc/

Code: http://www.polycephaly.org/fsbday/
(available under public domain)